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THE EVOLUTION OF SCHOOL MATHEMATICS

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The Madison Project has received a good deal of attention in the popular press as well as in professional publications. In this article, its Director describes the Project's "thrusts and probes" in mathematics education and reports on other current activities.

The Evolution of School Mathematics

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The way that children learn science and mathematics in our society is changing. These changes may represent the beginning of a profound alteration in American education. The present article is intended to comment on the apparent evolutionary changes in mathematics learning from the point of view of one of the participating "new curriculum" projects, namely the Madison Project of Syracuse University and Webster College.

The overall general picture of mathematics education for the next 20 or 30 years may be indicated somewhat by the report of the Cambridge Conference (sponsored by Educational Services Incorporated) which met at Radcliffe College during the summer of 1963. This conference was charged with the task of commenting on the directions that curriculum reform might take in the next three decades, with an attempt to distinguish some generally desirable directions from other generally less desirable directions.

The final report of the Cambridge Conference has not yet been released at the time of this writing. Nonetheless, it may be possible to make some remarks on the probable shape of this report. For one thing, it is gratifying that eminent research mathematicians and scientists were willing to devote an entire summer to an attempt to guide educational evolution. It is equally gratifying that the conferees have seen fit to entrust important portions of the mathematics curriculum to pre-college years. This reverses a trend that has been all too common in recent times: the tendency to say that "serious education" begins "at the secondary school," or "at the twelfth grade," or "upon entrance into college," or "after the freshman year," or, even, "when you enter graduate school." Many educators have been writing off more and more years at the start of a person's life, until we have found ourselves threatened with the loss of more productive years at the beginning of our lives, through educational lethargy, than we could ever hope to regain in our later years through advances in medicine, nutrition, and safety. The Cambridge Conference has reversed this trend. They have assigned such important topics as rigorous calculus, and serious linear algebra, to the domain of pre-college study. This is encouraging.

Evolution, however, is a tricky business. If you merely sit and watch it happen, you deserve what you will get, but you won't like it. If, at the opposite extreme, you try to plan evolution carefully and in detail, you thereby assign to yourself the task of

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predicting the future, and you will find yourself unable to fulfill this assignment with reasonable satisfaction.

The scientists and mathematicians meeting at Cambridge were aware of the need to guide evolution, yet of the simultaneous impossibility of making a detailed map of the unknown territory that lies spread out ahead of us along the positive direction of the time axis.

Clearly, this is a situation where we must make thrusts and probes into the darkness, and shape our plans according to the apparent results of these thrusts and probes.

Making such "thrusts and probes," and reporting the results to educators, scientists, and mathematicians, is one of the four major activities of the Madison Project, and one of the ways in which it attempts to contribute to rational guidance of educational evolution.

Reporting "Thrusts and Probes"

For example, what happens when you try to teach the arithmetic of signed numbers to first or second graders? The Madison Project has made a "probe" in this direction, and has reported, *via* film and written article, upon the outcome. (See the film *Kye's Arithmetic*, the film "Education Report," and the article *Math Takes a New Path*—all available from the Project at Webster College, St. Louis 19, Missouri. A booklet entitled "Arithmetic With Negative Digits" is in preparation. Cf. also the films "A Lesson with Second Graders," "First Lesson," and "Second Lesson.")

What happens when you let children in grades two through eight work with Cartesian coordinates? The Project has made an extensive thrust in this direction, and reported the results to the profession via a long sequence of films, including "A Lesson with Second Graders," "Second Lesson," "Circles and Parabolas," "Graphing an Ellipse," and "Weights and Springs," and also via various written materials.

What happens when you let children in grades four through seven work with algebraic identities and proofs of algebraic theorems? The Project has probed in this direction, and reported to the profession via tape recordings (especially tape recording number D-1), and via many films (especially "Second Lesson," "Axioms and Theorems," and "Theorem: $\square + \square = 2 \times \square$").

What happens when you let fifth, sixth, and seventh graders work with the algebra of matrices? (Cf. the films "Matrices," "Solving Equations with Matrices," and "Complex Numbers via Matrices.")

What happens when you try to teach a rigorous theory of limits (in the case of limit of a sequence) to eighth graders? The Project's first thrust in this direction is reported in the new film entitled "Limits."

What can you do to relate science and mathematics in grades five through eight? Some Project "thrusts" in this direction are reported in the films "Weights and Springs," "Average and Variance," "Velocity and Acceleration," "Forces and Vectors," and "Education Report."

Making such "thrusts and probes" is one of the four basic Project activities. From this, future curriculum planners can gain some guidance concerning the unknown world of future education and educational possibilities.

Enlarging the School Repertoire

A second Project activity is to attempt to move new mathematics into the standard repertoire. We believe this is also a matter for evolution. A new textbook series that included signed numbers in grades one and two, calculus in grade nine, and linear algebra in high school would prove unteachable, and would remain unadopted. The world isn't ready yet.

However, the world can be made ready! We can introduce signed numbers gradually into the elementary and even primary grade repertoire. We can introduce matrices into the fifth or sixth grade repertoire. We believe in building readiness for the child before each new plunge; it is equally

important to build readiness for the school and for the teacher.

Besides films, and the book *Discovery in Mathematics* (available from Addison-Wesley, Reading, Massachusetts), the Project's main effort in introducing new mathematical experiences into the school repertoire is contained in a "packaged" course for the in-service education of elementary teachers. This course is for group self-study by teachers (grades K–8), and is undergoing initial trials this fall. It makes use of both films and written materials, and assigns an active role to the teachers in their attempts to explore what can be done with new mathematical experiences for children.

Devising Creative Mathematical Experiences

The third task of the Project, and perhaps the task on which the others all rest, is to devise creative mathematical experiences for children. For example, we can play a modified game of Tic Tac Toe on Cartesian coordinates with second and third graders. The children learn to plot points accurately, they come to think of "mathematics" as something enjoyable, and they form habits of trying to see patterns and to devise shrewd and original strategies.

Again, we can ask four children independently to guess the length of the room. We record their data, and compute the average and variance. We pass out 6-in. plastic rulers, get four independent measurements, and compute the average and the variance. We repeat, using meter sticks. We repeat, using a surveyor's tape measure. The children enjoy this, get algebraic and arithmetical practice, and gain insight into the nature of measurement and the estimation of the amount of variability in a set of numbers.

Similarly, we can devise games based upon the ideas of implication, contradiction, and uniqueness (an adaptation of David Page's "Hidden Numbers" game).

Some of these experiences that children can have with mathematics are described in *Discovery in Mathematics;* others are described in the book *Matrices, Functions, and Other Topics* (available from the Project); and also in various films, (especially *A Lesson with Second Graders, First Lesson, Second Lesson, Weights and Springs, Average and Variance, Introduction to Logic, Circles and Parabolas, Graphing an Ellipse, Experience with Angles, Experience with Area, Matrices, Solving Equations with Matrices, Complex Numbers via Matrices, Velocity and Acceleration*, and others).

Toward a "Theory of Instruction"

From our seven years of effort in developing suitable creative mathematical experiences for children, we have developed the beginnings of what Bruner has called a "theory of instruction." Partial accounts of this "theory of instruction" have appeared elsewhere (see references). It is, in fact, an essential aspect of Project work. Teaching cannot deal in *topics*, such as "quadratic equations" or "matrix algebra" or "inference schemes" or "linear graphs." The mere mention of a "topic" such as this does not serve to define adequately the kind of experience the child will have. Hence, it does not define what he will learn. To discuss educational experiences on a level as abstract as merely mentioning "topics" is sometimes dangerously attractive, but cannot prove fruitful.

What is required, for meaningful discussion, is a fuller specification of the actual experience that the child will have. When we say that he will play the Project's modified version of Tic Tac Toe on Cartesian coordinates, for example, we have been far more explicit in defining the actual experience that we plan to provide, especially when we can show via films exactly what we have in mind.

In designing creative mathematics experiences for children, the Project tries to observe certain general principles:

1. "Large" mathematical ideas will usually be broken down into a sequence of

"smaller" ideas. This must be done, however, in a way that reveals the deeper structure of the mathematical reality, and not in a way that disregards or conceals this structure. The way in which this sequence of "smaller" ideas is worked out is a distinctive Project feature, and differs markedly from the usual practice with "programmed instruction."

2. Before a task is presented to a child, he is given considerable experience with the general techniques and concepts that will be involved. We call this "readiness building."

3. Notwithstanding the preceding point, it yet remains true that the overall pace of moving ahead with Project materials is very much faster than is usually the case in mathematics instruction. Probably most of the time is saved by a virtual absence of exposition, and by the fact that practice in one matter is always obtained in the course of exploring a new matter, and never by repetitive drill.

4. The experiences are structured in such a way that the student *wants* to do what we would like to see him do. Thus, to return to the modified Tic Tac Toe game, it is unnecessary in this game for the teacher to advocate precision in locating points. The student *himself* wants to be precise, for carelessness can destroy the strategy by which his team hopes to win.

5. The mathematical experiences should be as informal as possible, and should leave the child a large degree of autonomous control.

6. The selection of concepts and techniques with which the child is becoming familiar is largely restricted to really fundamental concepts and techniques, such as rational number, variable, open sentence and truth set, Cartesian graphs, function, implication, contradiction, uniqueness, linearity, transformation, matrix, axioms and theorems, extension of number systems, limit of a sequence, rate of change, slope, etc.

7. Each concept or technique should be "invented" or "discovered" by the child as he seeks to solve some kind of problem. For example, the concept of "prime" could emerge from the task of factoring integers. The definition of a prime would *not* be a starting point, but would rather be a summing up of what the child was discovering. Every significant mathematical concept or technique began as a "tool" for use in attacking some kind of mathematical problem. Only later were they transmuted to "definitions" and miscast as "starting points." We want the child to come upon these concepts in this same sense, as "tools" for tackling some mathematical task.

8. In each mathematical experience, the child should have an active role, rather than a passive one.

9. In each mathematical experience, there should be some "patterns" lurking beneath the surface, so that the child has a chance to make some interesting discoveries.

10. Each experience should be appropriate to the age and interests of the child. In general, for example, we are finding first graders to favor the independent exploration of material objects; fifth graders are "intellectuals," and enjoy extended abstract discussions in which they play a creative and active role; seventh and eighth graders are often "engineers," and prefer to study velocity and acceleration, to study the breaking strength of yarn, etc. (This is, needless to say, a vastly over-simplified discussion of the tortuously emerging changes in a child's interests and points of view as he grows.)

By making probes into the unknown potential of mathematics learning and reporting the results to the profession (especially via films), by working to introduce new portions of mathematics into the school repertoire, by devising creative school experience with mathematics, and by shaping a theory of instruction, the Madison Project seeks to give rational guidance to the evolutionary changes in school mathematics which appear to lie ahead.

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Gleason, Andrew M., Moise, E. E., et al., "Goals for School Mathematics," The Report of the Cambridge Conference on School Mathematics, Houghton-Mifflin, Boston, 1963.

2. Tape Recordings

Tape Recording No. D-1, "$\square + \square = 2 \times \square$," presents a classroom lesson with fifth graders, proving an algebraic theorem from a set of axioms, available from The Madison Project, Webster College, St. Louis, Missouri.

3. Films

Films are available from The Madison Project, Webster College, St. Louis, Missouri.

A Lesson with Second Graders, First Lesson, Second Lesson, Circles and Parabolas, Graphing an Ellipse, Matrices, Solving Equations with Matrices, Complex Numbers *via* Matrices, Weights and Springs, Forces and Vectors, Average and Variance, Velocity and Acceleration, Education Report: The New Math, Kye's Arithmetic, Limits, Theorem: $\square + \square = 2 \times \square$, plus others.

4. Films and Written Materials Combined

Supplementary Modern Mathematics, In-Service Course for Teachers, by the Madison Project staff; available from the Project.

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